

COMPACTED MATHEMATICS

CHAPTER 5

NUMBER SENSE

TOPICS COVERED:

- **Divisibility Rules**
- **Primes and Composites**
- **Prime Factorization**
- **Greatest Common Factor (GCF)**

What is an emirp number? It is a prime number that turns into another prime number when it is reversed. For example, 13 and 31 or 17 and 71. Did you notice that emirp is prime spelled backwards?

How about a palindromic number? It is a number that reads the same backwards and forwards. For example 111,111 or 2,867,682. Did you know that if you take any number and add it to the same number reversed (for example, 18 and 81), continuing the process as necessary, you sooner or later end up with a palindromic number? $68+86 = 154$ $154+451 = 605$ $605+506 = 1111$ (Yes!)

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|----------|-------------------------|-------|
| Activity | Jokes and Quotes | NAME: |
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| |
|---|
| Teacher: Where is your homework? |
| Student: I lost it fighting this kid who said you weren't the best teacher in the school. |
| If you plan for a year, plant a seed. If for ten years, plant a tree. If for a hundred years, teach the people. When you sow a seed once, you will reap a single harvest. When you teach the people, you will reap a hundred harvests. - Kuan Chung |
| Teacher: If $1+1+2$ and $2+2=4$, what is $4+4$? |
| Student: That's not fair! You answer the easy ones and leave us with the hard one! |
| The student came home from school with a long face. His dad asked, "What's wrong, son?" The boy replied, "The math test results came back today, Dad, and the teacher gave you a failing grade." |
| On arriving home from school, a little boy announced, "My math teacher is crazy." "Why?" his mother asked. "Yesterday," he said, "she told us five is four and one; today she is telling us that five is three plus two." |
| Little Tommy was in the first grade. One day, he came home and his mother asked: "Well, Tommy, what did you learn in school today?" "In math, I learned the three and three make seven." "But that's not correct," his mother said. "Well, then I guess I didn't learn anything." |
| Student: Wish I had been born 1,000 years ago! |
| Teacher: Why is that? |
| Student: Just think of all the history that I wouldn't have to learn! |

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| Activity 5-1 | Divisibility Rules | NAME: |
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Divisibility Rules

| | |
|---|---|
| 2 A number is divisible by 2 if the ones digit is even. | 3 A number is divisible by 3 if the sum of its digits is divisible by 3. |
| 4 A number is divisible by 4 if its last two digits are divisible by 4. | 5 A number is divisible by 5 if it ends in 0 or 5. |
| 6 A number is divisible by 6 if it is divisible by both 2 and 3. | 9 A number is divisible by 9 if the sum of its digits is divisible by 9. |
| 10 A number is divisible by 10 if its last digit is 0. | |

Circle the numbers that are divisible by 2.

| | | | | |
|-----|-----|------|------|-------|
| 34 | 58 | 67 | 90 | 241 |
| 324 | 243 | 432 | 423 | 234 |
| 196 | 825 | 4374 | 9701 | 65250 |

Circle the numbers that are divisible by 3.

| | | | | |
|-----|-----|-----|-----|------|
| 48 | 75 | 76 | 77 | 78 |
| 761 | 762 | 763 | 764 | 765 |
| 46 | 51 | 913 | 834 | 7085 |

Circle the numbers that are divisible by 4.

| | | | | |
|-----|-----|-----|-----|-----|
| 934 | 924 | 944 | 954 | 964 |
| 732 | 742 | 752 | 762 | 772 |

Circle the numbers that are divisible by 5.

| | | | | |
|-----|-----|------|------|-------|
| 354 | 355 | 375 | 380 | 385 |
| 650 | 605 | 506 | 560 | 1056 |
| 325 | 608 | 5280 | 8542 | 49104 |

Circle the numbers that are divisible by 6.

| | | | | |
|-------|-----|------|------|-------|
| 78 | 62 | 3054 | 5553 | 24718 |
| 69300 | 762 | 765 | 96 | 104 |

Circle the numbers that are divisible by 9.

| | | | | |
|------|------|------|------|------|
| 377 | 378 | 387 | 837 | 827 |
| 4876 | 5876 | 5976 | 9567 | 5796 |

Circle the numbers that are divisible by 10.

| | | | | |
|-----|-----|-----|-----|-----|
| 100 | 75 | 23 | 60 | 108 |
| 120 | 245 | 250 | 380 | 387 |

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| Activity 5-2 | Divisibility Rules | NAME: _____ |
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| 1. | The Southlake Carroll Marching Band is getting ready to perform at halftime of the football game. With 216 musicians, can the marching band form equal rows of 3? of 4? of 5? of 6? of 9? | |
| 2. | True or false: All numbers divisible by 5 are also divisible by 10. | |
| 3. | True or false: All numbers divisible by 10 are also divisible by 5. | |
| 4. | True or false: All numbers divisible by 9 are also divisible by 3. | |

Determine whether the first number is divisible by the second.

| | | | | | | | | |
|-----|-----------|--|-----|------------|--|-----|-----------|--|
| 5. | 185; 5 | | 6. | 76,870; 10 | | 7. | 461; 1 | |
| 8. | 456; 3 | | 9. | 35,994; 2 | | 10. | 6,791; 3 | |
| 11. | 12,866; 9 | | 12. | 7,564; 4 | | 13. | 45,812; 9 | |

| | | |
|-----|---|--|
| 14. | A giant pizza is divided into 18 pieces. What are the different numbers of people you can divide it among so that there are no pieces left over? | |
| 15. | You and 8 of your friends have been saving money by recycling aluminum cans. You have made a total of \$58.86. Can you divide the money evenly among yourselves? How can you tell without dividing? | |
| 16. | What is the smallest number you find that is divisible by 2, 3, 5, 6, 9, and 10? | |

Complete the table. Answer yes or no for each box.

| | Number | Divisible by 2 | Divisible by 3 | Divisible by 4 | Divisible by 5 | Divisible by 6 | Divisible by 9 | Divisible by 10 |
|-----|--------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|
| 17. | 324 | | | | | | | |
| 18. | 475 | | | | | | | |
| 19. | 525 | | | | | | | |
| 20. | 600 | | | | | | | |
| 21. | 1234 | | | | | | | |
| 22. | 3951 | | | | | | | |
| 23. | 4230 | | | | | | | |
| 24. | 7803 | | | | | | | |
| 25. | 9360 | | | | | | | |
| 26. | 11,235 | | | | | | | |
| 27. | 15,972 | | | | | | | |
| 28. | 23,409 | | | | | | | |

| | | |
|-----|--|--|
| 29. | Marty said to Doc, "So we are going to travel back in time. What year did you set the DeLorean for?" Doc replied, "I can't remember exactly, but I do remember the following: If you divide the year by 2, you'll get a remainder of 1. If you divide the year by 3, 4, 5, 6, 7, or 9, you'll also get a remainder of 1." "What about 8? Do you also get a remainder of 1?" "No," said Doc. Marty then knew which year they were off to. Which year? | |
|-----|--|--|

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| Activity 5-3 | Primes and Composites | NAME: _____ |
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Erathosthenes, an ancient Greek mathematician, developed a method to determine prime numbers. His method for finding the 25 prime numbers between 1 and 100 is explained below. A **prime number** is a number with only 2 factors. **Composite numbers** have more than 2 factors.

1. Cross out 1. One is not prime because it only has one factor (1).
2. Circle the smallest prime number. What is it? _____ Cross out all multiples of this number.
3. Circle the next prime number. What is it? _____ Cross out all multiples of this number.
4. Circle the next prime number. What is it? _____ Cross out all multiples of this number.
5. Circle the next prime number. What is it? _____ Cross out all multiples of this number.
6. Circle all of the prime numbers.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Using your 100 Board, answer the following questions.

| | |
|--|--|
| 1. What is the smallest prime number that is greater than 30? | |
| 2. What is the smallest prime number that is greater than 50? | |
| 3. 5 and 7 are called twin primes because they are both primes and they differ by two. List all twin primes between 1 and 100. | |
| 4. Find 5 composite numbers in a row. | |
| 5. Why didn't we have to keep going and circle all multiples of 9? | |
| 6. Which of the primes 2, 3, 5, and 7 divide into 84? | |
| 7. There are four columns on the board that contain no primes. Find them and explain why these columns contain no primes. | |

Cross out the boxes containing composite numbers to discover the hidden message.

| | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|-----|----|----|----|----|
| D | P | I | R | V | I | M | P | S | K | S | O | Z | R | S |
| 7 | 6 | 2 | 8 | 19 | 11 | 12 | 60 | 3 | 9 | 14 | 59 | 35 | 11 | 37 |
| Q | A | R | M | E | S | D | M | I | V | H | I | N | E | A |
| 4 | 3 | 31 | 25 | 23 | 10 | 29 | 12 | 41 | 97 | 100 | 23 | 83 | 13 | 12 |
| B | U | R | T | T | F | O | A | I | C | T | R | O | R | S |
| 71 | 2 | 35 | 3 | 27 | 43 | 42 | 37 | 64 | 7 | 5 | 45 | 13 | 11 | 71 |
| N | E | U | M | A | S | F | G | O | R | K | E | Q | T | D |
| 9 | 14 | 69 | 32 | 17 | 87 | 48 | 75 | 20 | 19 | 9 | 97 | 8 | 27 | 57 |
| F | R | C | I | M | E | T | K | N | D | L | N | I | E | R |
| 67 | 2 | 16 | 89 | 18 | 7 | 12 | 9 | 17 | 73 | 67 | 49 | 59 | 29 | 83 |

| | | |
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| Activity 5-4 | Prime Factorization | NAME: _____ |
|--------------|----------------------------|-------------|

Determine whether each number is composite, prime, or neither.

| | | | | | |
|----|----|--|-----|---------|--|
| 1. | 18 | | 2. | 31 | |
| 3. | 1 | | 4. | 434 | |
| 5. | 97 | | 6. | 111,111 | |
| 7. | 57 | | 8. | 4,293 | |
| 9. | 73 | | 10. | 38 | |

Create a factor tree to find the prime factorization of each number. Write your answer using exponents when necessary.

| | | | | | |
|-----|-------|--|-----|-------|--|
| 11. | 280 | | 12. | 92 | |
| 13. | 900 | | 14. | 20 | |
| 15. | 35 | | 16. | 54 | |
| 17. | 96 | | 18. | 64 | |
| 19. | 30 | | 20. | 85 | |
| 21. | 108 | | 22. | 45 | |
| 23. | 88 | | 24. | 43 | |
| 25. | 78 | | 26. | 84 | |
| 27. | 1,024 | | 28. | 2,400 | |

Find the missing factor.

| | | | |
|-----|--|-----|--|
| 29. | $3^2 \cdot 5 \cdot \underline{\hspace{1cm}} = 315$ | 30. | $2^4 \cdot \underline{\hspace{1cm}} \cdot 7 = 1,008$ |
| 31. | $3^3 \cdot \underline{\hspace{1cm}} = 135$ | 32. | $2^2 \cdot 3^2 \cdot \underline{\hspace{1cm}} = 252$ |
| 33. | $5^2 \cdot \underline{\hspace{1cm}} = 275$ | 34. | $3^2 \cdot 5^2 \cdot \underline{\hspace{1cm}} = 2,475$ |

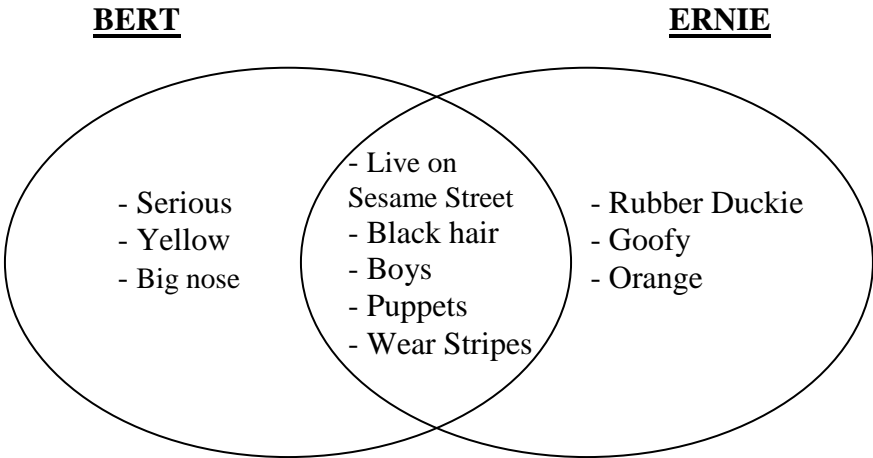
Create a factor tree for each of the composite numbers listed below. Write the prime factorization using exponents.

| | | | | | |
|-----|-----|--|-----|-----|--|
| 35. | 210 | | 36. | 128 | |
| 37. | 324 | | 38. | 68 | |
| 39. | 132 | | 40. | 53 | |
| 41. | 394 | | 42. | 87 | |
| 43. | 235 | | 44. | 420 | |

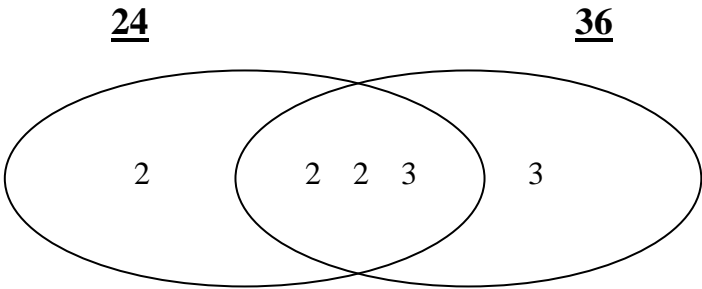
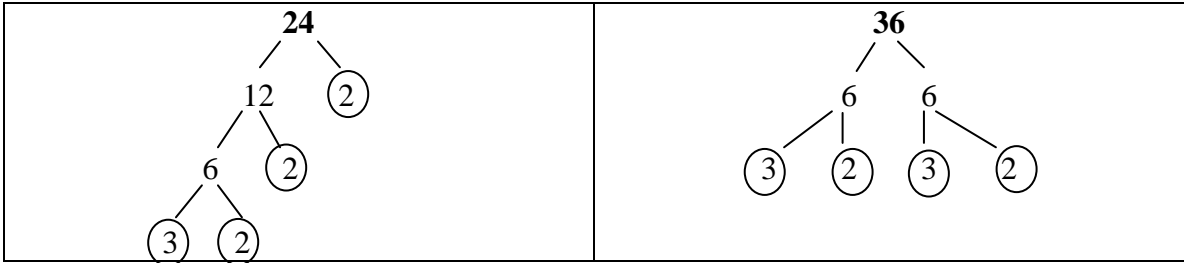
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|-----|--|--|
| 45. | What is the maximum number of prime numbered dates in any two consecutive months? | |
| 46. | I am a two-digit prime number. The number formed by reversing my digits is also prime. My ones digit is 4 less than my tens digit. What number am I? | |

| | | |
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| Activity | Venn Diagrams | NAME: |
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Venn diagrams can be used to tell similar and different characteristics about two or more items. The Venn Diagram below compares and contrasts Bert and Ernie. Other ideas for introduction: Elmo, Cookie Monster, SpongeBob, Barney, etc.



Venn diagrams can be combined with a factor tree to find LCM's, GCF's, and for simplifying fractions.



GCF = multiply the factors that were in common = $2 \cdot 2 \cdot 3 = 12$
 LCM = multiply all of the factors in the diagram = $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$
 To simplify the fraction $\frac{24}{36}$ look at the factors outside the common area = $\frac{2}{3}$
 $24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$ $36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$

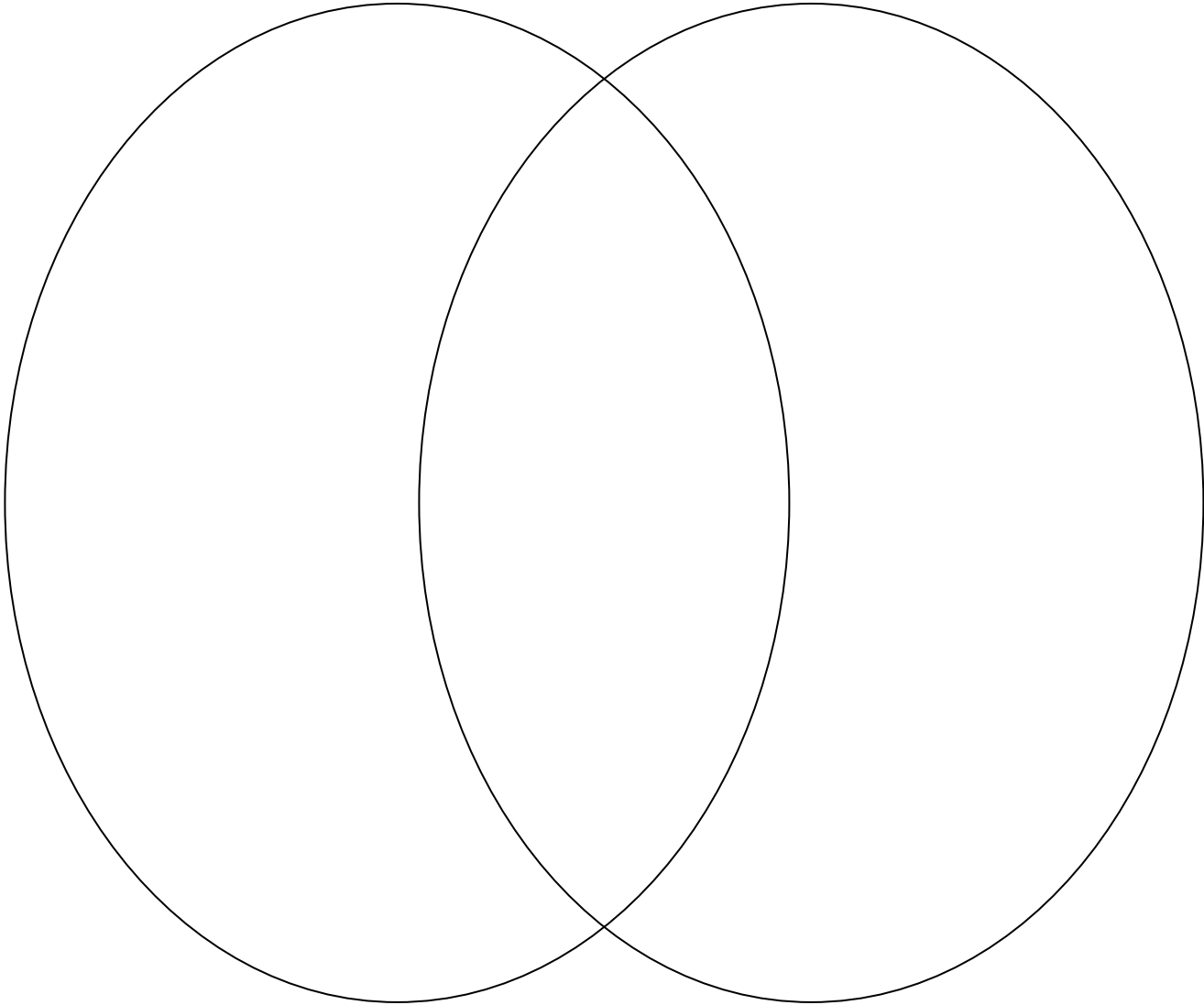
Activity

Venn Diagrams

NAME:

HARRY POTTER

SPONGEBOB



| | | |
|--------------|-------------------------------------|-------------|
| Activity 5-5 | Greatest Common Factor (GCF) | NAME: _____ |
|--------------|-------------------------------------|-------------|

Factor means maker, doer, or performer. Two factors multiplied together make a product. Just as factories produce products, so too do factors produce products.

The greatest of the common factors of two or more numbers is called the **greatest common factor (GCF)**.

There are two ways to find the GCF. The first is to list all of the factors of each number.

Example 1 Find the GCF of 30 and 42 by making a list.

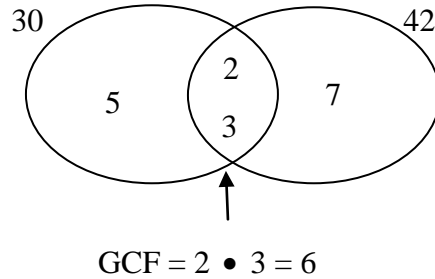
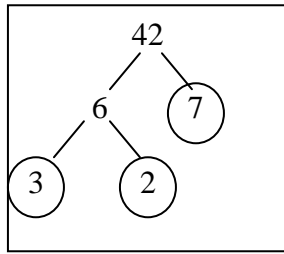
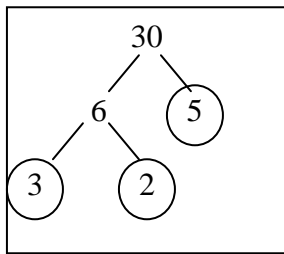
Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

The common factors are 1, 2, 3, and 6. The greatest common factor is 6.

The second method is to use factors trees and a Venn diagram.

Example 2 Find the GCF of 30 and 42 using factor trees.



Find the GCF by making a list.

| | | | | | |
|----|--------|--|----|------------|--|
| 1. | 12, 16 | | 2. | 18, 24 | |
| 3. | 20, 16 | | 4. | 8, 4 | |
| 5. | 30, 36 | | 6. | 35, 49 | |
| 7. | 32, 40 | | 8. | 12, 18, 24 | |

Find the GCF with factor trees and a Venn diagram.

| | | | | | |
|-----|--------|--|-----|------------|--|
| 9. | 39, 26 | | 10. | 12, 20 | |
| 11. | 35, 28 | | 12. | 40, 56 | |
| 13. | 36, 48 | | 14. | 24, 30 | |
| 15. | 54, 72 | | 16. | 28, 42, 56 | |

17. The GCF of two numbers is 850. Neither number is divisible by the other. What is the smallest that these two numbers could be?

18. The GCF of two numbers is 479. One number is even and the other number is odd. Neither number is divisible by the other. What is the smallest that these two numbers could be?

19. Ms. Wurst and Mr. Pop have donated a total of 91 hot dogs and 126 small cans of fruit juice for a math class picnic. Each student will receive the same amount of refreshments. What is the greatest number of students that can attend the picnic? How many cans of juice will each student receive? How many hot dogs will each student receive?

| | | |
|--------------|-------------------------------------|-------|
| Activity 5-6 | Greatest Common Factor (GCF) | NAME: |
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Find the GCF by making a list.

| | | | | | |
|----|------------|--|----|------------|--|
| 1. | 12, 13 | | 2. | 15, 25 | |
| 3. | 15, 18 | | 4. | 27, 54, 81 | |
| 5. | 16, 24, 56 | | 6. | 21, 30, 44 | |

Find the GCF with factor trees and a Venn diagram.

| | | | | | |
|-----|-------------|--|-----|-------------|--|
| 7. | 48, 72 | | 8. | 90, 150 | |
| 9. | 88, 110 | | 10. | 85, 51 | |
| 11. | 75, 90, 120 | | 12. | 45, 70, 120 | |

13. The GCF of two numbers is 871. Both numbers are even and neither is divisible by the other. What is the smallest that these two numbers could be?

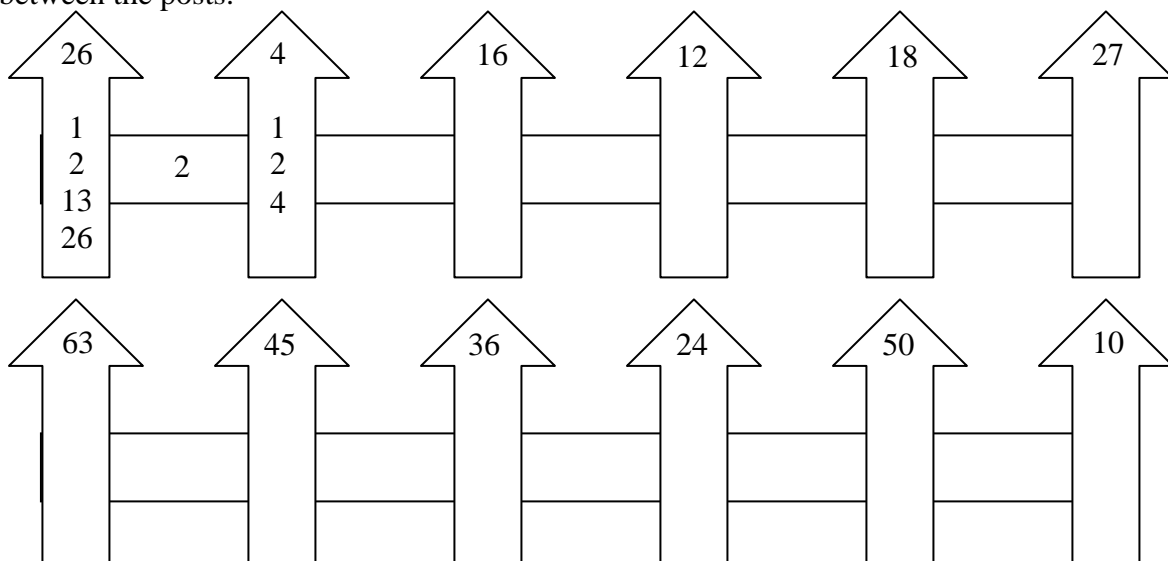
14. Ms. Wurst and Mr. Pop have donated a total of 90 hot dogs and 126 small cans of fruit juice for a math class picnic. Each student will receive the same amount of refreshments. What is the greatest number of students that can attend the picnic? How many cans of juice will each student receive? How many hot dogs will each student receive?

15. Mr. Mangham has a certain number of cookies. They can be divided evenly among 9 students. They can also be divided evenly among 6 students. What are two possibilities for the number of cookies?

16. A band of pirates divided 185 pieces of silver and 148 gold coins. These pirates were known to be absolutely fair about sharing equally. How many pirates were there?

Factor Fences

Write the factors of each composite number below it on the fence post. Write the GCF in the rungs between the posts.



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| Activity 5-7 | Juniper Green | NAME: |
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Juniper Green – Round 1

- Rules of the game:
1. Two players play at a time. The first player selects an even number.
 2. On each turn, a player selects any remaining number that is a factor or a multiple of the number just selected by his or her opponent.
 3. The first player who cannot select a number loses.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Juniper Green – Round 2

- Rules of the game: The rules this time are very similar to the first game, except you and your partner are now working together. Try to stay alive as long as possible by crossing out as many numbers as possible. The game is over when a player cannot select another number.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

| | | |
|----------|----------------------------------|-------|
| Activity | The World's Easiest Quiz? | NAME: |
|----------|----------------------------------|-------|

| | | |
|-----|--|--|
| 1. | How long did the Hundred Years War last? | |
| 2. | Which country makes Panama hats? | |
| 3. | From which animal do we get catgut? | |
| 4. | In which month do Russians celebrate the October Revolution? | |
| 5. | What is a camel's hair brush made of? | |
| 6. | The Canary Islands in the Pacific are named after what animal? | |
| 7. | What was King George VI's first name? | |
| 8. | What color is a purple finch? | |
| 9. | What country do Chinese gooseberries come from? | |
| 10. | How long did the Thirty Years War last? | |
| 11. | The maker does not want it; the buyer does not use it; the user does not see it. What is it? | |
| 12. | Before Mt. Everest was discovered, what was the highest mountain on Earth? | |
| 13. | In what year did Christmas and New Year's fall in the same year? | |
| 14. | How many times can you subtract 5 from 25? | |
| 15. | Even if they are starving, natives living in the Arctic will never eat a penguin's egg. Why not? | |